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Research in basic mathematics has been supported by the past three years under NASA Grant NGR-44-005-010. Primary effort has been to solve the normal Moore conjecture, one of the outstanding problems in point set topology.

Progress made is indicated in preprints of papers which have been earlier submitted or are being herewith submitted. At least two papers remain in progress and will be submitted within the next six months.

A summary of the results obtained is offered herein.

In a paper (to appear in the Pacific Journal), authored jointly with J. N. Younglove, several conjectures due to Heath and Grace were proved to be such that, if true, then sufficient to establish that each normal, separable Moore space is metrizable. This opens the problem to attack from a more general setting. Details are included in the paper "On normality and pointwise paracompacts". Preprints have earlier been submitted to the granting agency.

In a paper "Some conditions equivalent to normality" (submitted to the Duke Journal of Mathematics) it is pointed out that earlier results included in a paper in that journal were in error and correction is offered which establish conditions that are equivalent to normality. In this paper, success is achieved in the effort to consider the problem from a functional viewpoint. This gave rise to the investigations of the possibility of defining a reasonable integral in Moore spaces which would substitute the property that each closed set be a G_δ (inner limiting set) for the usual property of local compactness. It was hoped that, by developing a measure theory for a general topological space without the condition of local

compactness, and effort might be made to case such problems as differential equations into that general setting, but with solution easier to obtain than in a metric space.

This did not meet with success, inasmuch as it was established that such a space, (if it is to be topological) must necessarily be a sum (perhaps uncountable) of hedgehogs; i.e., the sum of dendron-like structures. The space so reduced does not appear of sufficient interest to investigate properties of measure or integration which can be defined on it.

What does seem possible is that proximity spaces (which are not topological, but give rise to a topology) might well lend themselves to a development of measure and integration with solutions to applied problems realized with less effort by casting the problem into that setting. Efforts in this direction are only now initiating but appear promising.

Further research was accomplished which extended results due to E. E. Grace and were contained in the paper "Weak peripheral properties imply covering properties". This paper has been submitted to the Proceedings of the London Math. Society and includes results which reduce the metrization problem of Moore spaces to consideration of nowhere dense subsets.

Submitted herewith are preprints of a paper which generalizes results due to A. H. Stone and approaches the metrization problem using notions of weight and density character of the space. This paper is now being submitted to Mathematika and offers additional notrization theorems for developable spaces.

In the paper "Completeness in developable spaces," a new notion of sequential completeness is introduced and thus is investigated from the

viewpoint of establishing metrization of normal pointwise paracompact developable spaces.

It seems entirely possible now that it will be possible to complete pointwise paracompact developable spaces. This would make it considerably more of interest to know that each normal Moore space is pointwise paracompact for it would then be known that each such space may be embedded in a complete Moore space and this would grant the existence of a dense metric subspace of the original space.

Questions raised in each of the preprints earlier submitted remain of interest. Additionally, it seems to be more important to know whether each countably paracompact Moore space is normal. It has been decided now by Mr. Robert C. Briggs, a student working under the direction of the principal investigator, that there is a countably paracompact T_3 space which is not normal. Additional results concerning the equivalence of countable paracompactness, normality, hereditary countable paracompactness and inverse limit systems of countably paracompact spaces have been established by Mr. Phillip Zenor, another student working under the direction of the principal investigator. These results will be included with the dissertation of the students with preprints submitted this agency beforehand, although support here is only indirect.